# Time Dilation, as Manifested in Special Relativity, Explained with Euclidean Geometry

Part 1

By Bob Duhamel

Before starting, I have to point out that you will find many explanations about special relativity in various venues. These explanations fall into two categories. The first comes from physicists who use math to explain special relativity. These are almost always correct. The second comes from laypeople who explain special relativity by showing how our perspective is affected by the speed of light. These are always wrong. Light *will* take longer to travel farther distances (propagation delay). However, that does not result in the effects of special relativity. Such explanations usually work when objects are traveling apart. However, they fail when objects travel toward each other. Correct explanations work regardless of the direction of travel. Special relativity results from how different frames of reference align when observers move relative to each other in four-dimensional spacetime. Propagation delay must be added to the effects of special relativity to calculate when and where an event will be seen.

This essay explains the effects of special relativity using Euclidean geometry, which the average educated person can understand. This is possible because special relativity is a consequence of the alignments of frames of reference in four-dimensional spacetime where time is the fourth dimension.

Using Euclidean geometry to visualize the effects of special relativity is not new. Hermann Minkowski developed such a system shortly after Einstein published his theory. However, it is difficult for the uninitiated to understand Minkowski's system. I will show a system based on simple Euclidean geometric principles that the average educated person can understand. This system has some difficulties that require careful examination to understand the results. For example, the numbers don't add up in this first essay unless we include length contraction in the calculations. Since we don't address length contraction in this first essay, we must assume that length contraction takes place. In the second essay, length contraction is shown to be an obvious consequence of relative motion, but without careful examination, events don't seem to align visually in space and time. However, modifying the graphics to give a useful visual representation of time dilation is easy.

In this first essay, we will look at imaginary everyday events. By extrapolating these events into a spacetime continuum with time as a fourth dimension, we will see that time dilation is inevitable when observers move relative to each other.

## Preceding special relativity

Before Einstein developed his theory of special relativity, physicists found conflicts between observations and the expected outcomes of certain experiments. The most famous was the Michelson-Morley experiment.[1] At the time, physicists assumed that if you shined light beams in different directions, the light would appear to travel at different speeds due to the Earth's motion through space. Michelson and Morley didn't get the expected result. No matter how they rotated their apparatus, the light appeared to be going at the same speed.

Hendrik Lorentz and George FitzGerald postulated that the Michelson-Morley apparatus contracted in the direction it was moving through space. They theorized that as matter moves through space, it moves through a substance called luminiferous aether. Just as a soap bubble distorts in the wind, matter moving through the aether should "flatten" in the direction of motion. They developed the following formula to quantify this length contraction. Today, this is called the Lorentz factor or gamma.

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where:

| | | |
|---|---|---|
| $\gamma$ | = | Lorentz factor (gamma) |
| $c$ | = | Speed of light in a vacuum (empty space)[2] |
| $v$ | = | The velocity of an object |

Even though modern theories eliminate the aether, the Lorentz factor now applies to special relativity.

Many special relativity formulas incorporate the Lorentz factor. However, the reciprocal of the Lorentz factor (alpha) applies to Euclidean geometry, such that we can use it to visualize time dilation and length contraction in special relativity.

$$\alpha = \sqrt{1 - \frac{v^2}{c^2}}$$

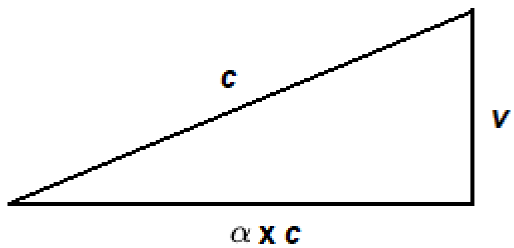
The reciprocal of the Lorentz factor or alpha

[1] https://en.wikipedia.org/wiki/Michelson-Morley_experiment

[2] No insult intended, but during my live lectures, I discovered that many people think I'm talking about a vacuum cleaner when I talk about a vacuum. In scientific parlance, a vacuum is a place devoid of matter, such as outer space.

**The Lorentz factor and right triangles**

The Lorentz factor alpha relates to a right triangle in Euclidean geometry. The length of the hypotenuse represents the speed of light (*c*), and the opposite represents the object's speed (*v*). Alpha represents the relative length of the adjacent compared to the hypotenuse; multiply the length of the hypotenuse by alpha to get the length of the adjacent.

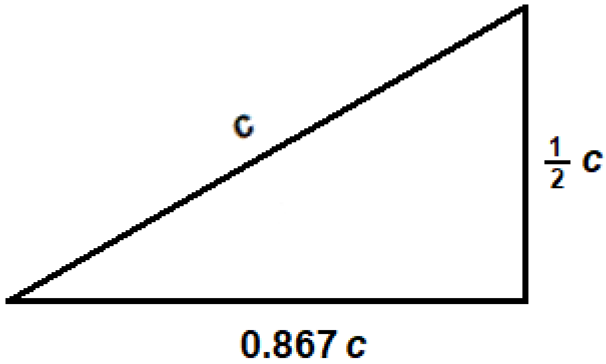



A right triangle where the length of the hypotenuse represents the speed of light (c) and the opposite represents the velocity of an object (v). Multiply the speed of light by alpha to get the length of the adjacent (base of the triangle). Compared to the hypotenuse, the length of the adjacent is proportional to time dilation and length contraction in special relativity.

In special relativity, alpha represents the length contraction or time dilation factors. If you plug the velocity of an object into the alpha formula, the result is the factor to apply to the object's length at rest to get its length at speed. Likewise, alpha tells how much slower time passes for a moving object from a stationary point of view.[3]

For example, *c* is always 299,792,458 meters per second. So if *v* is 149,896,229 meters per second (one-half *c*), alpha is 0.866602504. Here is a right triangle representing that calculation.

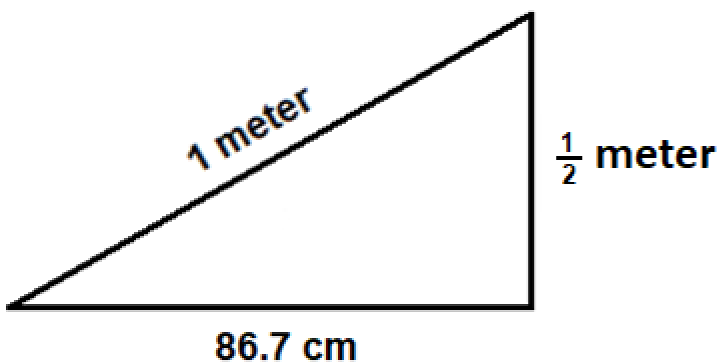
[3] This is the reciprocal of time dilation. Time dilation (lengthening of time, quantified by gamma) measures how much longer a moving clock takes to reach a particular time seen from a stationary frame of reference. Alpha measures how much slower that clock "ticks."




This right triangle represents alpha, where *v* is one-half the speed of light. The result is an Alpha of 0.867. Multiply alpha by *c* to get the length of the adjacent.

This demonstrates that the Lorentz factor can be plotted using familiar Euclidean geometry and a right triangle. In special relativity, the value of the adjacent is the factor to apply to the length of an object at rest to find its length at speed. Therefore, if we substitute the hypotenuse's length with the object's length at rest, the adjacent becomes the object's length at speed.

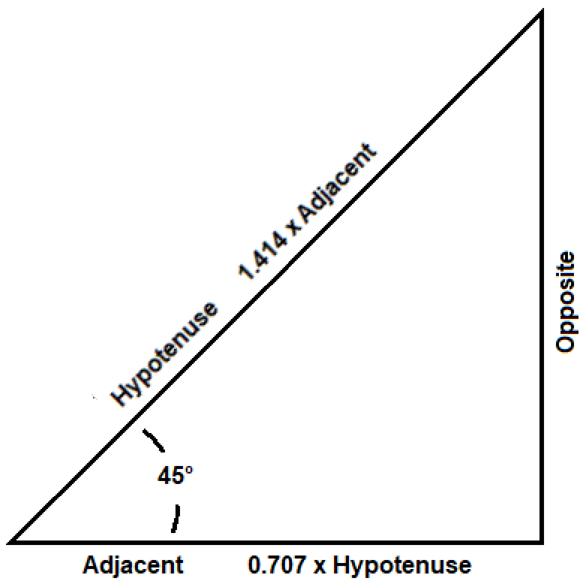
Assume an object is one meter long at rest. Traveling at ½ the speed of light, the object is 86.7 centimeters long (the length is contracted only in the direction of travel).




Substituting the length of the hypotenuse with the length of an object at rest, the adjacent becomes the object's length at speed. For example, an object one meter long at rest becomes 86.7 centimeters long (0.867 meters) when traveling at ½ the speed of light. In the above diagram, ½ meter represents a velocity of ½ the speed of light.

This calculation and triangle also apply to time dilation in the same way. If an observer could watch a clock pass by at half the speed of light, he or she would see that clock ticking at only 86.7 percent of its ticking speed at rest.

The following illustration uses a speed of 70.7 percent of the speed of light. This is because if we apply that speed to the geometry, we get a 45-45-90 triangle.




A 45-45-90 triangle has one 90-degree angle and two 45-degree angles.

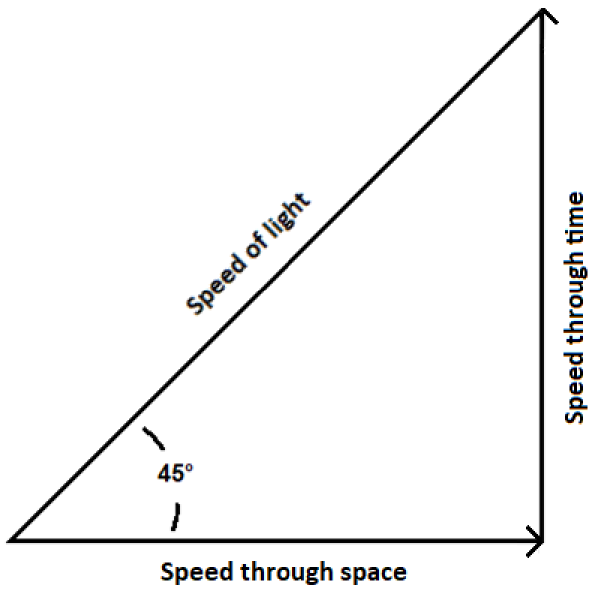
The hypotenuse of a 45-45-90 triangle is 1.414 (the square root of 2) times the length of the adjacent or the opposite. The adjacent and the opposite are the same lengths and are 0.707 times the length of the hypotenuse—the adjacent and the hypotenuse forming a 45-degree angle.

At 70.7 percent of the speed of light, gamma is 1.414, and alpha is 0.707. A stationary observer would see the length of an object moving at 70.7 percent of the speed of light shortened in the direction of motion to 70.7 percent of its length at rest. Likewise, the stationary observer would see a clock moving at 70.7 percent of the speed of light ticking at 70.7 percent of the speed that it ticks at rest. Using a velocity of 0.707*c*, we can quantify the effects of special relativity without calculating gamma or alpha; we already know the easily remembered solutions.

**Minkowsky diagrams**

Once Einstein developed his theory of special relativity, Hermann Minkowski developed a method to illustrate four-dimensional spacetime using two-dimensional graphs. To accommodate this, Minkowski incorporates all three space dimensions into a single dimension. His graphs plot time on the vertical axis and space (all three dimensions) on the horizontal axis. Minkowski uses standard vector analysis to compare a stationary observer (traveling through time only) to a moving observer (moving through space and time).

Minkowski's diagrams show the speed of time and the speed of light with equal magnitude. Therefore, a (massless) object traveling through space at the speed of light for a certain amount of time is represented by two nose-to-tail arrows (vectors[4]) of equal length placed at a right angle. These vectors become the adjacent and opposite of a triangle where the hypotenuse forms a 45-degree angle and has a length of 1.414 times the length of either of the other two sides. Therefore, an object traveling at the speed of light is represented by a line angled 45 degrees to the time or space axes.




The speed of light represented with two vectors (horizontal and vertical arrows) representing the speed of light through space and the speed of light through time, respectively.

I have mentioned Minkowsky's methods because they are already widely used in physics. However, Minkowsky's methods are unintuitive to those of us who are accustomed to regular distance/time graphs. Fortunately, a normal distance/time graph is not only more intuitive for most people but also demonstrates *why* time dilation and length contraction must occur if you add a fourth dimension (time) to our familiar three-dimensional space.

[4] A vector is a visual element used to represent quantities that cannot be represented with a single number. Typically, a vector represents a direction and a magnitude. For example, an aircraft traveling at 100 km per hour on a course of 270 degrees cannot be quantified with a single number. However, for example, it can be represented on a drawing surface by an arrow with a length of 100 mm pointing to the left, assuming that up represents zero degrees on a compass.

**Let's get started**

Now that we have established the relationship of special relativity to Euclidean geometry, let's imagine some everyday experiences and see how they lead us to special relativity.

The following illustrations show how frames of reference change when we move through space. The math that physicists apply to special relativity also applies to these effects. Special relativity is a complex topic, so I will be precise in my language and repeat myself occasionally.

**Einstein's assumptions**

Einstein made some crazy assumptions about space and time that are true. He based his theories on the mathematics of the Lorentz transformation, which can be applied to a continuum with four dimensions. We will examine this as a space-time continuum, where time is the fourth dimension.

We can measure our movements using a coordinate system when we move about in space. We can assign a grid in three dimensions where any point in space can be defined in that grid. First, of course, we must begin with an arbitrary starting point, but from there, we can describe any point in space relative to that starting point. For example, a particular point could be described as "right 3 meters, forward 2.5 meters, and up 1 meter." It's like navigating a big city. To get from Point A to Point B, you start at your current location, go so many streets in one direction, then so many streets in another. You may be able to go in a straight line between the points, but street-by-street descriptions based on a grid of streets are the usual way to navigate a city.

There is another way to define your coordinate system. If you are moving, you have a coordinate system that is aligned with the direction you are moving. Those coordinates are right, left, forward, backward, up, and down. Let's say someone else is going in a different direction. His or her coordinate system is aligned at an angle to yours. Their right, left, forward, and backward are different directions than yours.[5]

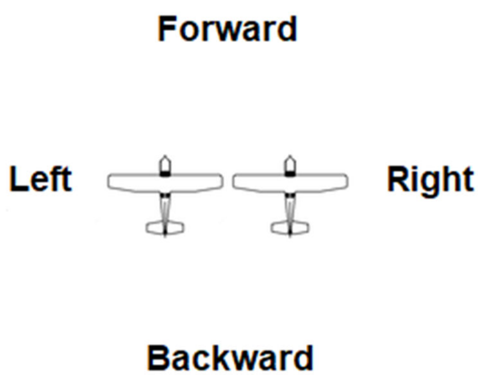
This is where Einstein bases his theory of relativity. Two people (observers, as Einstein called them), moving in different directions, have coordinate systems

[5] You are already familiar with this—"is that my right or your right?"

aligned at an angle to each other. Without external references outside their coordinate systems, Einstein imagined how these observers would see each other's motion. Then, he imagined that time is another dimension that follows the same rules as the three space dimensions. The only difference between space and time is that we can control our movement through space; however, we always move at a constant "speed" through time. So he questioned how two observers would see each other move through time as one or both observers moved through space.

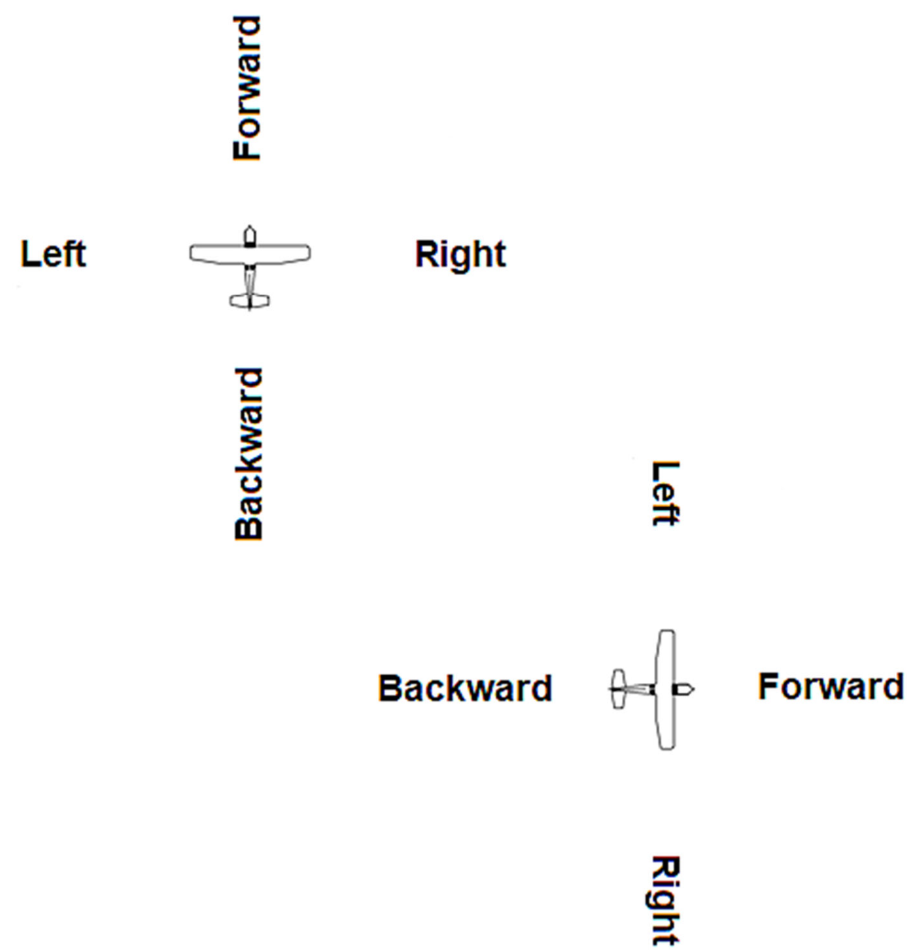
First, let's look at how two moving observers see each other move through space.

Let's say you have two airplanes traveling together. Right, left, forward, and backward are the same directions for each; they have identical frames of reference.




Two airplanes traveling in the same direction have identical frames of reference.

Now, let's say one airplane makes a 90-degree turn. The two airplane's frames of reference are now rotated 90 degrees to each other.



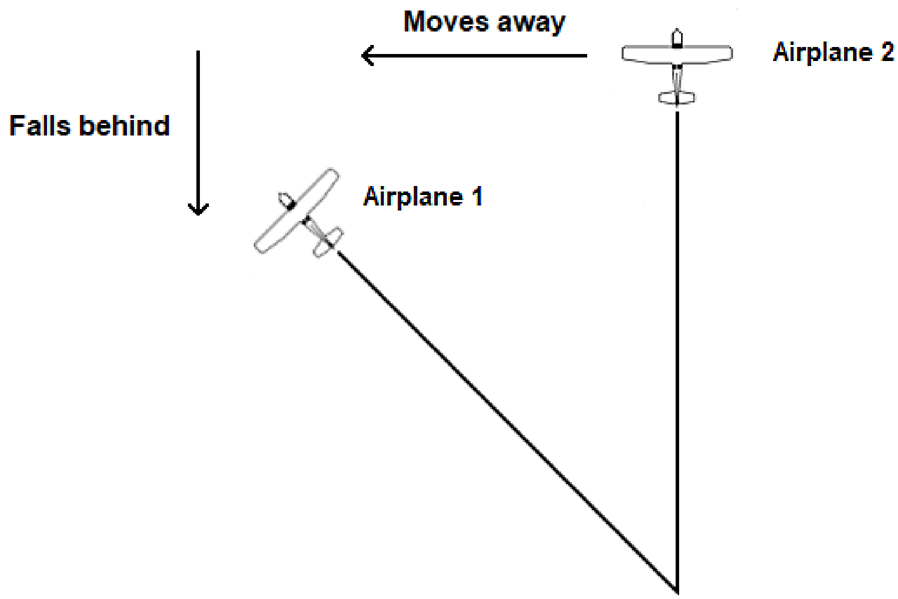

Two airplanes traveling in different directions have frames of reference angled to each other.

Either airplane's frame of reference is aligned with the direction it travels and is independent of the other airplane's frame of reference.

Let's assume those airplanes, Airplane 1 and Airplane 2, travel together at a constant speed of 100 kilometers per hour. The airplanes are flying between two layers of cloud, so observers aboard the airplanes cannot see the ground or the sky; they have no external clues to tell them what direction or how fast they are going. The only visual references they have are each other.

At some point, airplane 1 turns left and travels on a new course angled 45 degrees to the original. How do observers in these airplanes see each other moving?

The observer in Airplane 2 looks to his or her left and sees Airplane 1 moving away. He or she also sees airplane 1 falling behind.



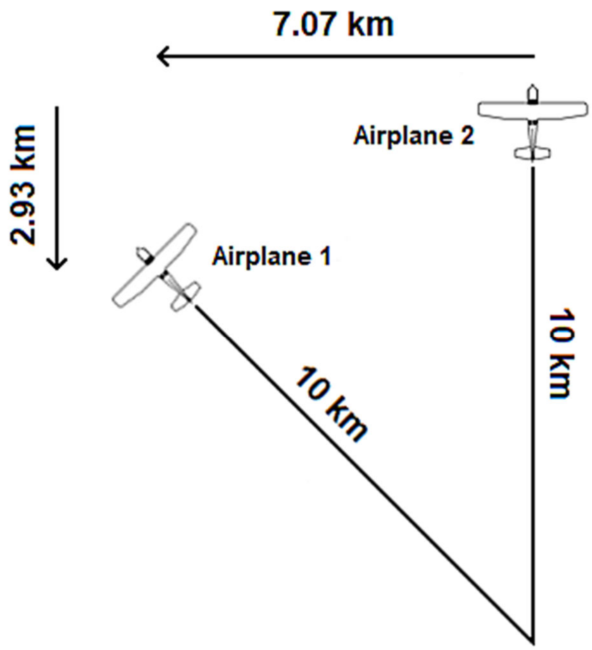

When Airplane 1 changes course, the observer in Airplane 2 sees it move away and fall behind.

Both airplanes still travel at 100 km/h but in different directions. For Airplane 1 to remain abreast of Airplane 2, Airplane 1 would have to speed up to about 141 km/h (100 km/h x 1.414).

What does Airplane 1 see?

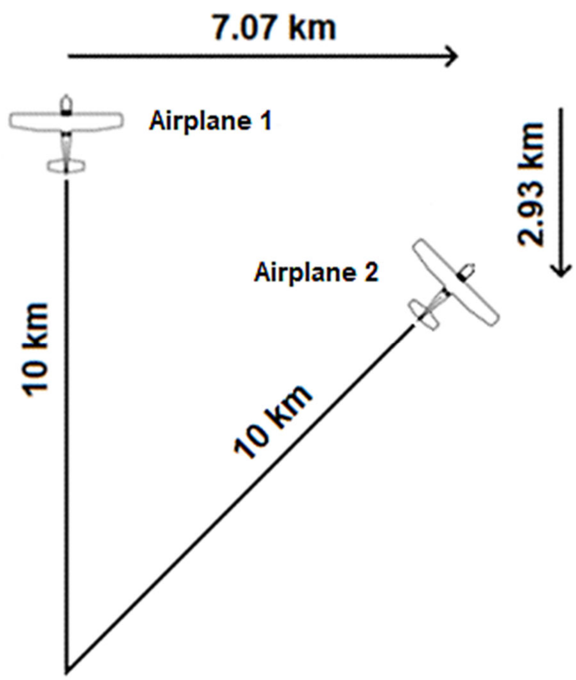
Let's assume that the observer in Airplane 1 didn't know he or she had turned. For all he or she knows, it is Airplane 2 that turned 45 degrees to the right. The observer in Airplane 1 looks to the right and sees Airplane 2 moving away. He or she also sees airplane 2 falling behind. Even though both airplanes travel at the same speed, each observer sees the other Airplane falling behind. It doesn't matter which Airplane turned. Each observer travels at the same speed but in different directions and sees the other airplane falling behind.

After six minutes, both airplanes have traveled 10 kilometers but in different directions. The observer in Airplane 2 looks to the left and sees that Airplane 1 has traveled to the left for 7.07 kilometers and has fallen behind by 2.93 kilometers.




After six minutes, from Airplane 2's frame of reference, Airplane 1 has moved away, to the left by 7.07 kilometers, and has fallen behind by 2.93 kilometers.

From Airplane 1's frame of reference, it appears that Airplane 2 has moved away to the right at a 45-degree angle. It also appears that airplane 2 has traveled to the right for 7.07 kilometers and fallen behind by 2.93 kilometers. Each Airplane has flown 10 kilometers, but to each observer, it appears the other Airplane has fallen behind by 2.93 kilometers.

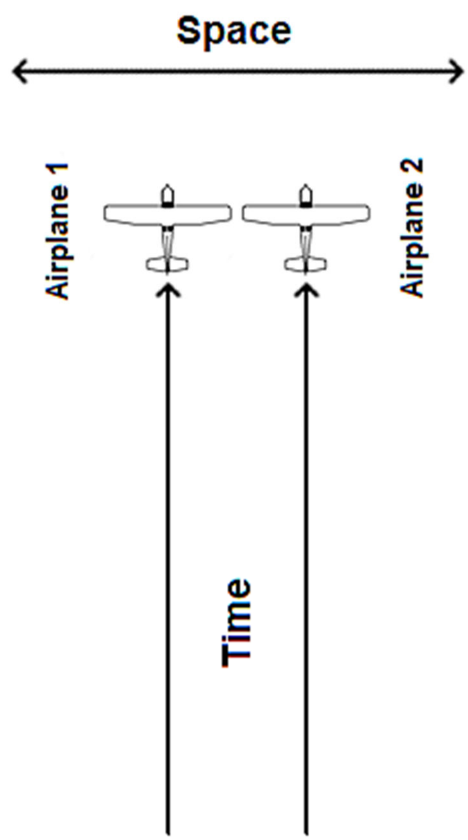



From Airplane 1's frame of reference, Airplane 2 has moved away to the right by 7.07 kilometers and has fallen behind by 2.93 kilometers.

Now, let's see what happens if we swap out one of the space dimensions and replace it with the time dimension. Time is just another dimension with no special consideration other than we move through time at a constant speed. Therefore, we can treat time exactly like any of the three dimensions of space.

Let's park the airplanes so they are no longer moving through space. Are they moving? Yes. Since everything constantly moves through time, the airplanes are moving through time.

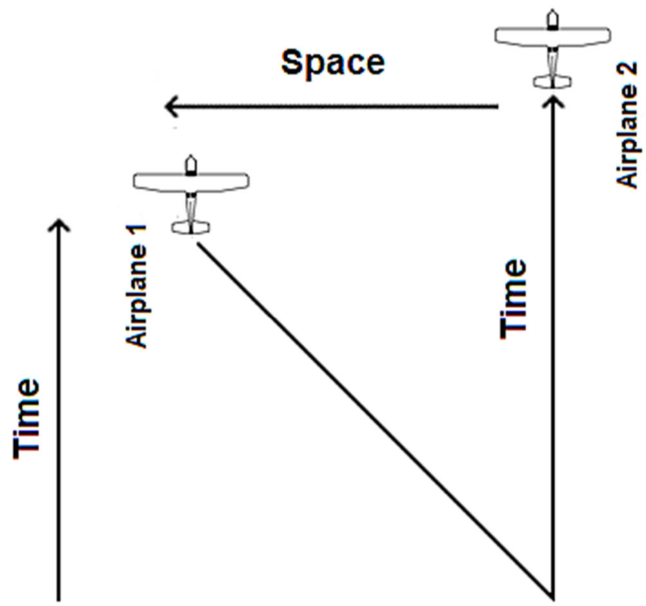
Let's start again with the planes now motionless in space but traveling together through time. In the following diagram, we find that time is one axis, and the other axis is space. We have consolidated all three dimensions of space into one axis simply because we can't illustrate all four dimensions on a flat plane. Nevertheless, we can make this illustration work if we only travel in two dimensions. So now we have the airplanes traveling from bottom to top through time. Traveling through space is shown by moving the airplanes left and right.




The airplanes are stationary, so they are not moving through space, but they are moving together through time.

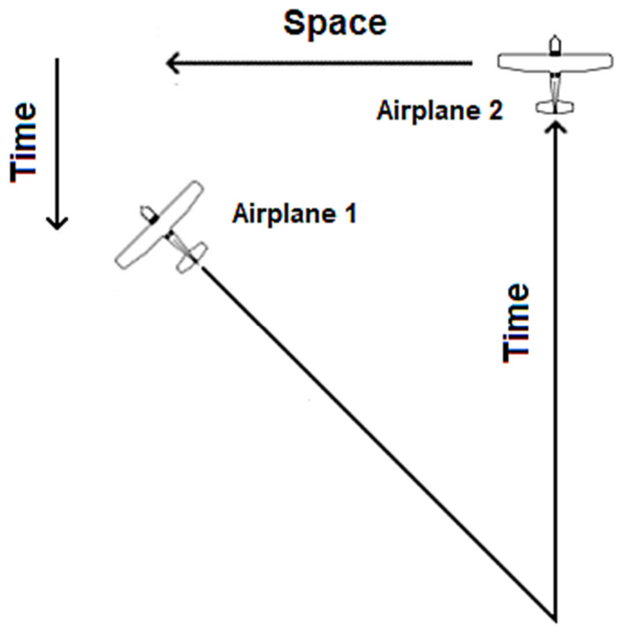
Now, let's put Airplane 1 on a dolly and push it sideways for some distance.

Here's where Einstein made a crazy assumption. The assumption is that time, being another dimension, must act the same as any of the three dimensions of space. In addition, time must have the same relationship to the three space dimensions that they have to each other. In the original scenario, when Airplane 1 moved away from Airplane 2, Airplane 1 changed its course through space. Consequently, Airplane 1 moved away from and fell behind Airplane 2. Now, we have swapped dimensions, and the airplanes travel together through time. So, when Airplane 1 is pushed away from Airplane 2, it must also change its course through space and time (spacetime).



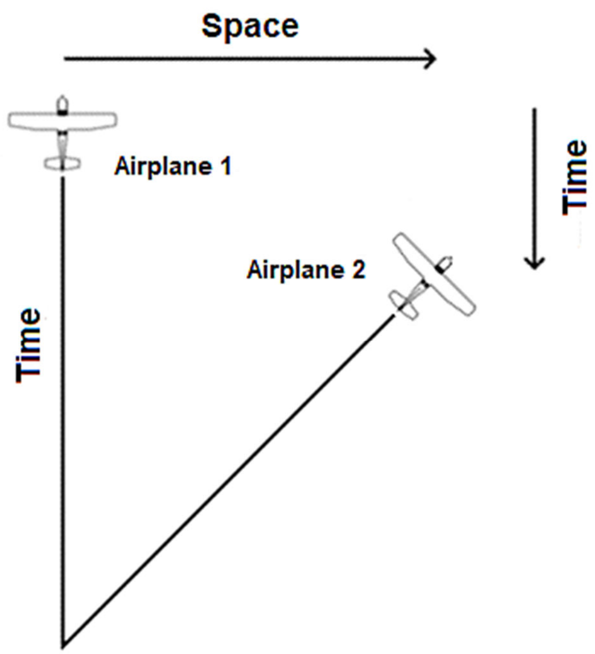

By moving, Airplane 1 changes course through spacetime.

In the original scenario, when Airplane 1 changed its course, it fell behind Airplane 2. In this new scenario—with time acting like space—when Airplane 1 changes its course through spacetime, it must fall behind Airplane 2 in time.




When Airplane 1 changes its course through space and time, from Airplane 2's frame of reference, Airplane 1 moves away from Airplane 2 in space and falls behind Airplane 2 in time.

Here's where Einstein made another crazy assumption. Airplane 2 travels through time, but Airplane 1 travels through space and time. Einstein assumed that both observers must perceive that they are stationary and that the other airplane is moving. Therefore, each observer must perceive that he or she is moving through time only and must see the other Airplane as the one moving through space and time. From Airplane 1's frame of reference, it is Airplane 2 that is moving away and falling behind in time.




From Airplane 1's frame of reference, it is Airplane 2 that is moving away and falling behind in time.

At this point, we assume that if either observer could see the clock on the instrument panel of the other Airplane, he or she would see the other Airplane's clock running slower than his or her own. This is time dilation.

Let's reiterate that because that is the essence of special relativity. While traveling together in space, when one Airplane moves away from the other in space, each observer sees the other Airplane falling behind in space. Likewise, when the airplanes move together in time when one moves away from the other in space, each observer sees the other Airplane fall behind in time.

To make a significant difference, you must travel at tremendous speeds. For example, to rotate your course through spacetime by 14 degrees, you must travel through space at 25 percent of the speed of light. To rotate your course through spacetime by 45 degrees, you must travel through space at 70.7 percent of the speed of light. If two observers diverge at 70.7 percent of the speed of light, each will see the other falling behind in time by 2.93 seconds for every ten seconds of travel. However, at ordinary speeds, the difference is too small to perceive.

This is the essence of time dilation. It may sound absurd, and Einstein never explained it quite this way. He described it mathematically. However, based on work by Lorentz and others, his math applies to a coordinate system with four dimensions where time is treated identically to the three space dimensions. Here, I have illustrated graphically what Einstein described mathematically.

Time dilation may be hard to fathom, but it has been observed to occur with subatomic particles traveling near the speed of light.[6] In addition, GPS satellites, moving at about 14,000 kilometers per hour relative to the Earth, have atomic clocks programmed to compensate for time dilation to keep them in sync with ground-based atomic clocks.

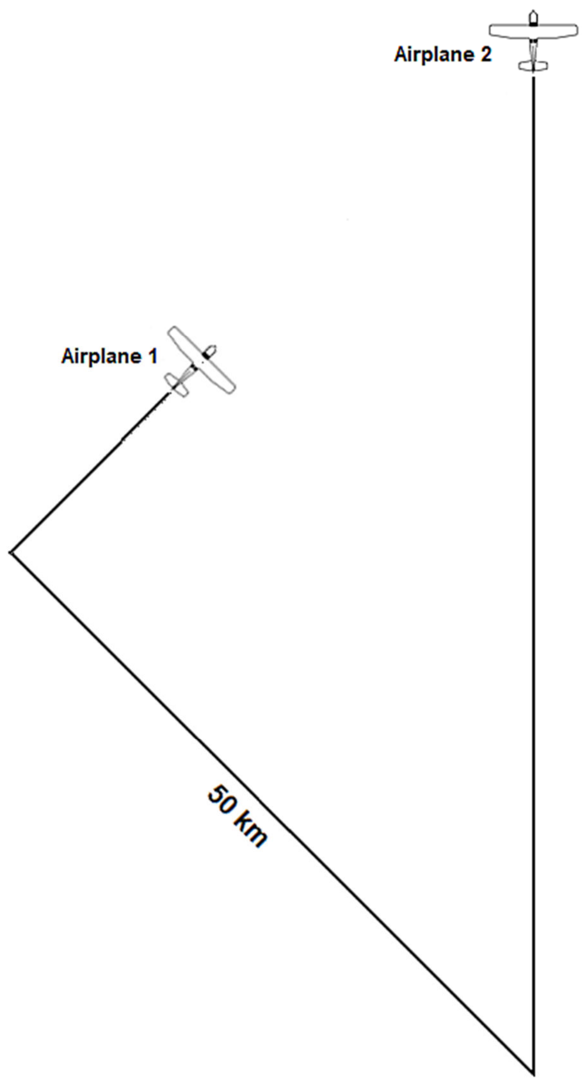
You may find some explanations about time dilation that say each observer sees the other observer's clock running slower because it takes longer for the light from each clock to reach the observers as they get farther apart. However, propagation delay is not the cause of time dilation. Time dilation results from the alignment of the different frames of reference. The propagation delay must be added to time dilation to calculate how each observer would see the other Airplane's clock. As the observers move away from each other, each observer would actually see the other airplane's clock running slower than time dilation predicts. When the airplanes move closer together, the effect of propagation delay reverses; the observers would see each other's clocks running faster than time dilation predicts.[7] In this illustration, we ignore propagation delay to simplify the example.

Now, let's see what happens when the airplanes come back together.

Suppose that in the original scenario, after 1/2 hour, Airplane 1 makes a 90-degree turn back toward the original course. Airplane 1 now travels along a course angled 45 degrees toward the original course. What happened to its frame of reference? It rotated clockwise. Where is Airplane 2 from Airplane 1's perspective now? Airplane 1 is now traveling toward Airplane 2 and sees Airplane 2 up ahead.

[6] The video, *Time Dilation, an Experiment with Mu-Mesons* (https://youtu.be/rbzt8gDSYIM), shows how the experiment was done.

[7] The stationary observer would see the approaching obsers's clock tiching faster than time dilation predicts but he or she would still see it ticking slower than his or her own clock.




After traveling for one-half hour (and 50 kilometers), Airplane 1 turns back toward the original course. Now, Airplane 2 appears to be ahead of Airplane 1.

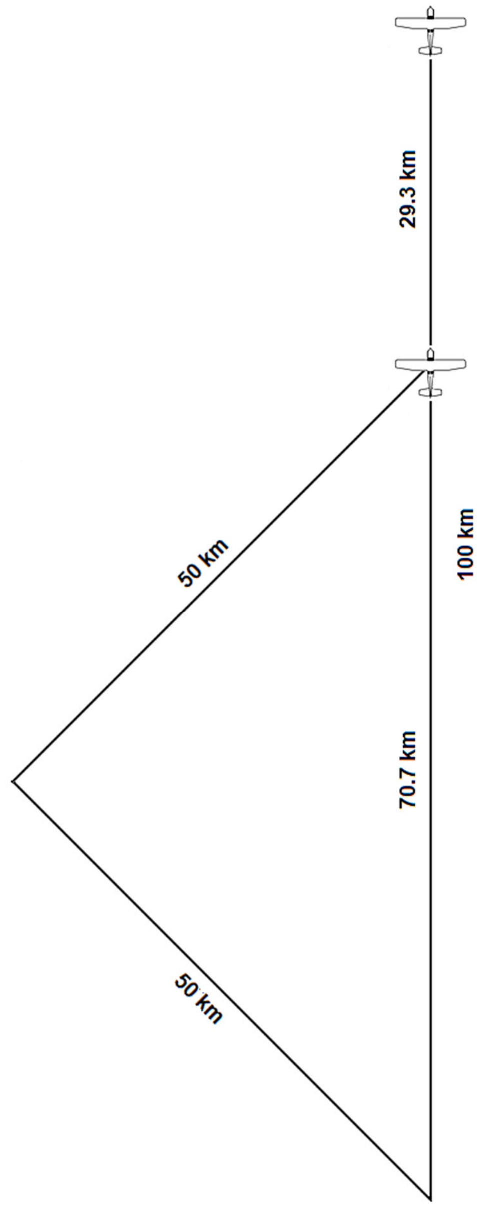
When Airplane 1's frame of reference rotates at the turnaround point, Airplane 2 goes from behind Airplane 1 to ahead of Airplane 1.

Let's stop and examine this for a moment. It is essential to understand how time dilation works. Imagine you are the observer in Airplane 1. As you travel away from Airplane 2, you must look back over your right shoulder to see Airplane 2 as it appears to have fallen behind. Think about what you will see as your airplane turns back toward the original course. As you turn, you will see airplane 2 sweep around until it is in front of you (at about the one o'clock position). Imagine it again.

Airplane 2 appears to go from behind to in front of you. Airplane 2 did nothing. It was *your* turn that changed *your* frame of reference.

After another 1/2 hour, airplane 1 intercepts the original course but is now 29.3 kilometers behind Airplane 2. As a result, Airplane 1 will now always see Airplane 2 about 30 kilometers ahead and can never catch up as long as both airplanes maintain a speed of 100 km per hour and Airplane 2 never changes its course.

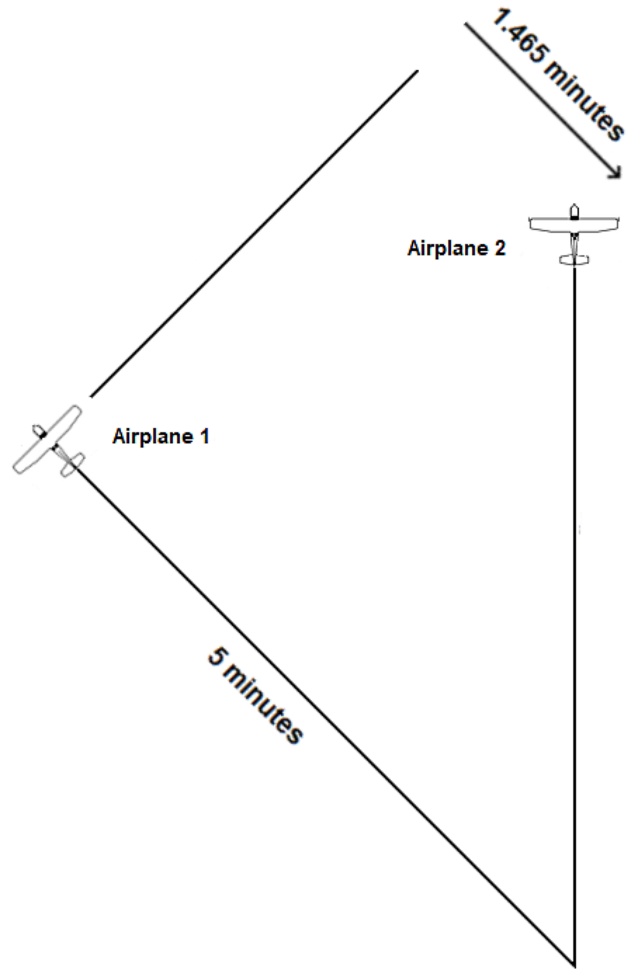
Of course, the observer in Airplane 2 now sees Airplane 1 about 30 kilometers behind. Airplane 2 traveled 100 kilometers, as did Airplane 1. However, having deviated from a straight course through space, Airplane 1 lost ground to Airplane 2.




If Airplane 1 returns to the original course after diverging for a half hour, it will lose about 30 kilometers to Airplane 2. Airplane 1 has traveled 100 kilometers in the hour it took for the total deviation but has only traveled 70.7 kilometers along the course that Airplane 2 has traveled.

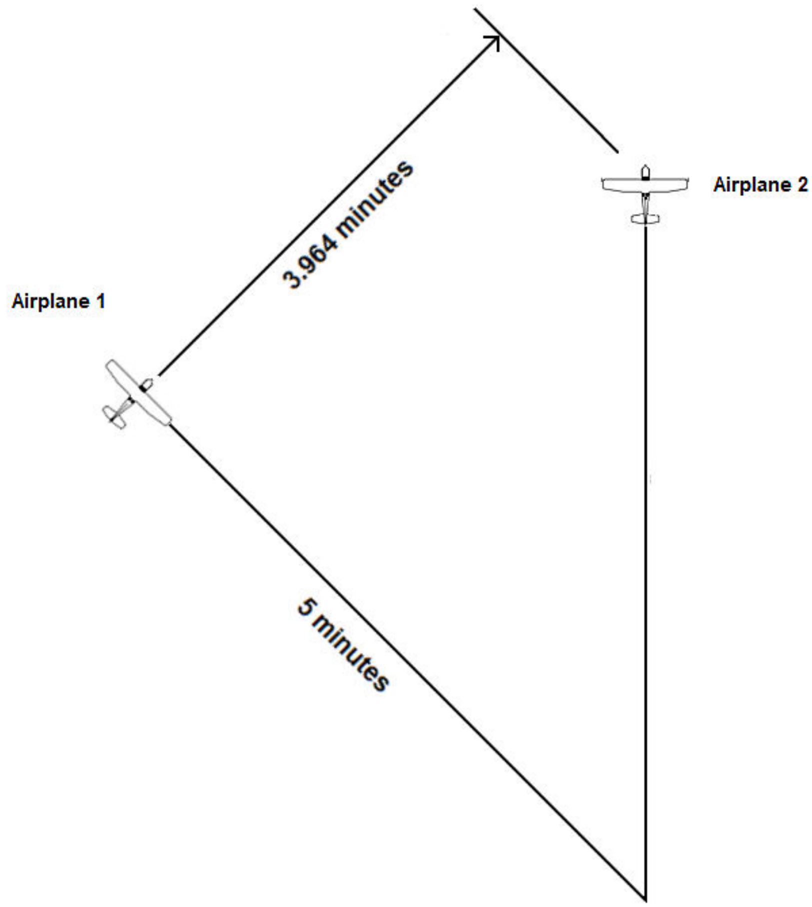
Now, let's get back to the new scenario.

Let's say Airplane 1 is pushed away from Airplane 2 at 70.7 percent of the speed of light for five minutes. At the end of five minutes, the observer in Airplane 1 will see that Airplane 2 has fallen behind in time by 1.465 minutes.




After traveling through space at 70.0 percent of the speed of light for five minutes, Airplane 1 sees that Airplane 2 has fallen behind in time by 1.465 minutes.

Now, let's push Airplane 1 back toward Airplane 2, again at 70.7 percent of the speed of light. When Airplane 1 reverses direction, the observer in Airplane 1 will see Airplane 2 ahead in time by 3.964 minutes.



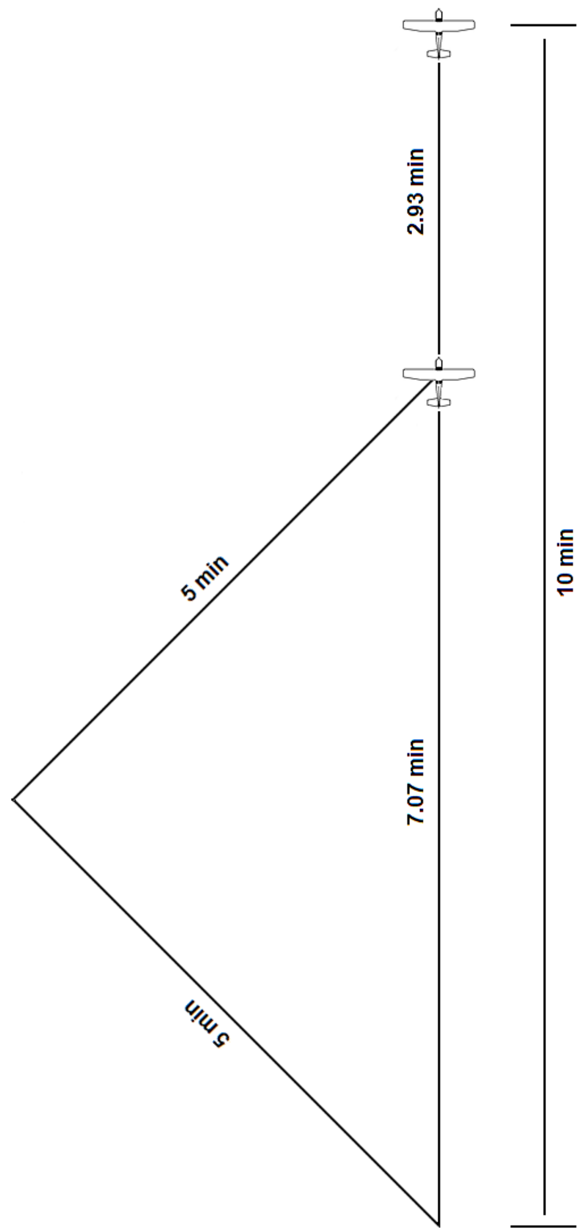

When Airplane 1 reverses course through space, the observer in Airplane 1 sees Airplane 2 move ahead in time. Airplane 2 Is now 3.964 minutes ahead of Airplane 1.

Let's stop and take a look at that again. When Airplane 1 reaches the turnaround point after traveling for five minutes, the observer in Airplane 1 sees that the clock in Airplane 2 has only ticked off 3.535 minutes. Of course, the observer in Airplane 2 sees the same phenomenon looking at Airplane 1's clock. The observer in Airplane 2 sees that the clock in Airplane 1 has only ticked off 3.535 minutes. Each observer sees that the other observer's clock is ticking slower than his or her own. This is what we expect from time dilation.

However, when Airplane 1 reverses course, the observer in Airplane 1 sees the clock in Airplane 2 jump ahead. Just as in the first scenario, where the observer in Airplane 1 saw Airplane 2 swing ahead in space, in the second scenario, the observer in Airplane 1 sees Airplane 2 swing ahead in time. Airplane 2 appears to zoom through 5.429 minutes, ending up 3.964 minutes ahead of Airplane 1.

Now, let's see what happens when Airplane 1 returns to its original position in space next to Airplane 2. The observer in Airplane 1 finds that Airplane 2's clock is

2.93 minutes ahead of the clock in Airplane 1. Both Airplanes have traveled through 10 minutes of time, yet Airplane 1 finds itself 2.93 minutes behind Airplane 2.




If Airplane 1 moves away from Airplane 2 at 70.7 percent of the speed of light and then returns taking 10 minutes for the round trip, Airplane 1 will fall behind Airplane 2 in time by 2.93 minutes. Airplane 1 traveled 10 minutes during its trip but only 7.07 minutes along Airplane 2's timeline.

This graphically illustrates how time dilation must occur when objects move relative to each other and compares it to an experience most people can relate to. However, it is only half the story.

Airplane 1 has clearly gone through a full 10 minutes of time in its frame of reference but has only gone through 7.07 minutes of time according to Airplane 2's frame of reference. Yet it seems that both airplanes' clocks register 10 minutes.

This is no different from the first scenario. In that case, both airplanes traveled 100 kilometers, yet Airplane 1 had only traveled 70.7 kilometers along the course taken by Airplane 2. If we look closer, we see we are comparing apples and oranges. To say Airplane 1 has traveled only 70.7 kilometers, we have to measure its progress from Airplane 2's frame of reference. The scenario helps us visualize special relativity but is not the whole story.

**The rest of the story is length contraction.**

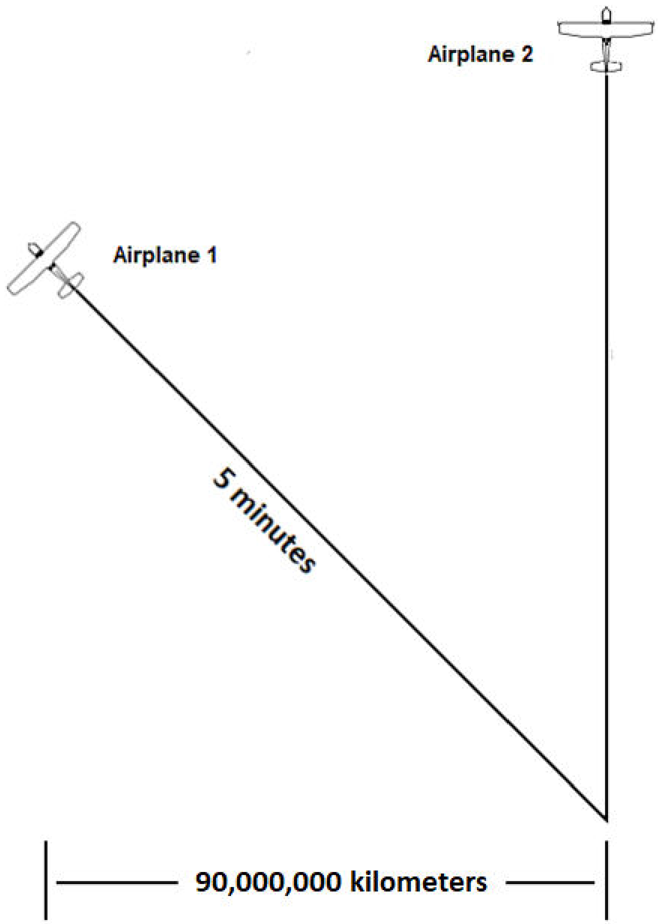
At this point, we need to accept length contraction on faith. Otherwise, we must enter a long discussion to explain the cause of length contraction (which we will do in part two of this essay). For now, let's accept that when an object moves as viewed from a stationary frame of reference, that object is contracted in the direction of motion. For example, if you see a 10-meter-long airplane fly by at 70.7 percent of the speed of light, it will appear flattened in the direction of travel to 7.07 meters long.

Of course, you can't tell who's moving. If you pass by a 10-meter-long airplane at 70.7 percent of the speed of light, that airplane will still be contracted to 7.07 meters. You can't tell if the airplane is moving if you are moving, or if you are both moving.

This doesn't only go for moving objects. It goes for space. If you are moving through space, it appears space is moving past you. That space is contracted in the direction of motion. If you travel to an object 100,000,000 kilometers away at 70.7 percent of the speed of light, it will seem that you traveled only 70,700,000 kilometers to reach it.

**Including length contraction**

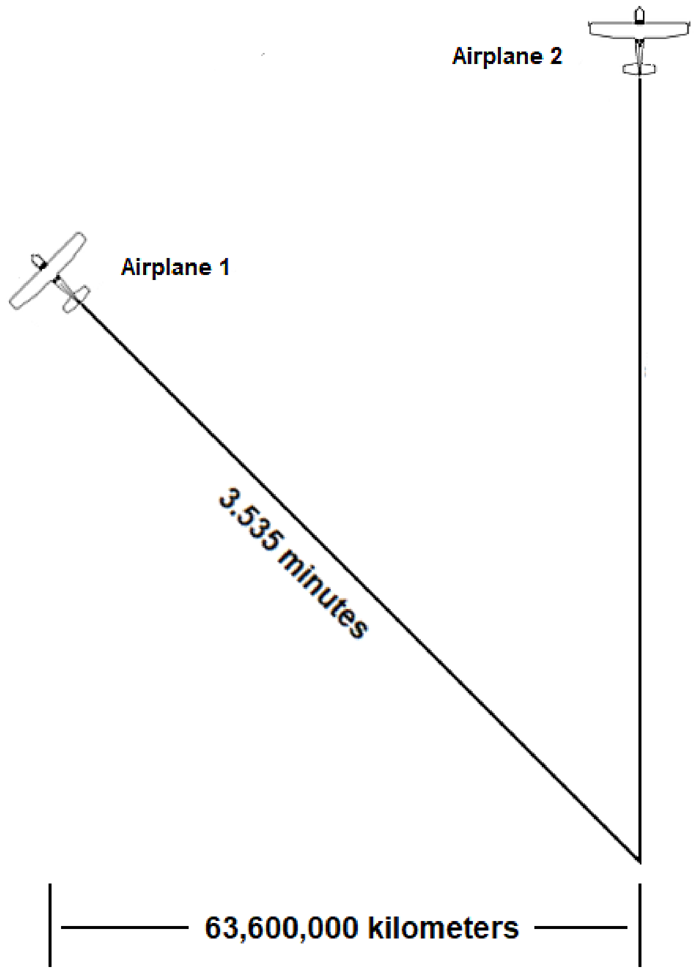
Let's replay the second scenario, but this time, when we push Airplane 1 away from Airplane 2, let's aim to reach a point in space 90,000,000 kilometers away, traveling at 70.7 percent of the speed of light. From Airplane 2's frame of reference, assuming the observer in Airplane 2 could see Airplane 1 throughout its trip without propagation delay, the trip will take five minutes.




**Airplane 2's Perspective**

From Airplane 2's frame of reference, Airplane 1 takes five minutes to reach an object 90,000,000 kilometers away at 70.7 percent of the speed of light.

However, the observer in Airplane 1 finds that it only takes 3.535 minutes to reach the destination. Why? Because the observer in Airplane 1 sees space going by at 70.7 percent of the speed of light. At this speed, space is contracted by a factor of 0.707, contracting 90,000,000 kilometers to 63,600,000 kilometers. Therefore, at 70.7 percent of the speed of light, Airplane 1 reaches the destination in only 3.535 minutes, according to its clock.



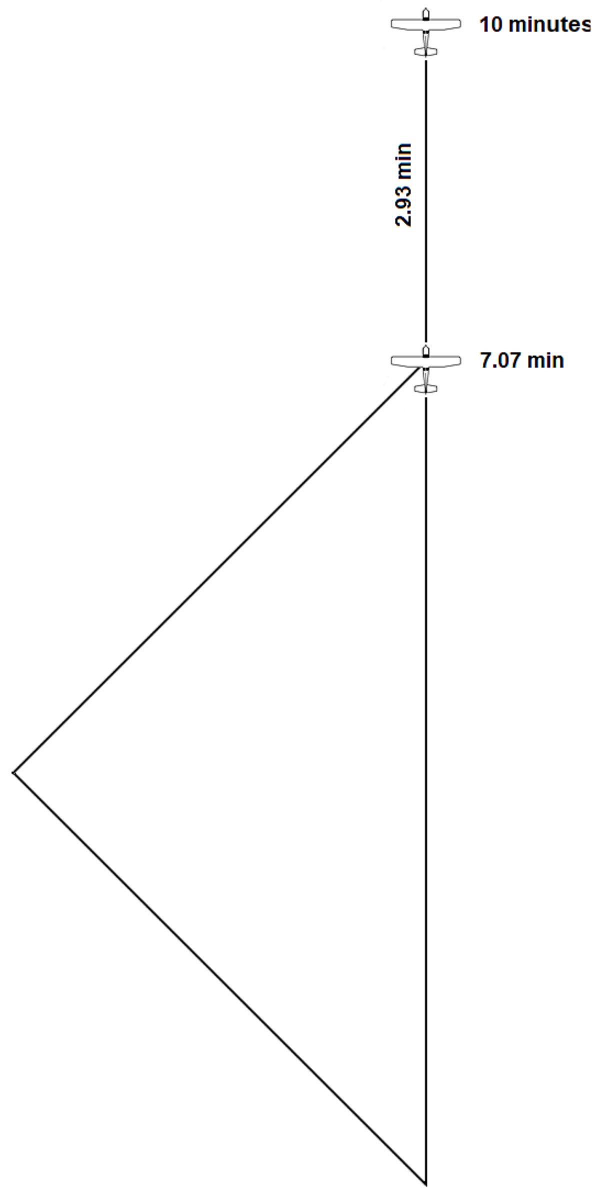

**Airplane 1's perspective**

Due to length contraction, Airplane 1 reaches an object 90,000,000 away in only 3.535 minutes, traveling at 70.7 percent of the speed of light. From Airplane 1's frame of reference, the distance is only 63,600,000 kilometers. (Note that this graph shows Airplane 1 moving even though the observer in Airplane 1 should perceive Airplane 1 as stationary. This is to illustrate that the observer in Airplane 1 will perceive his or her motion through space by observing objects in space passing by.)

If the observer in Airplane 2 could observe the trip without propagation delay, he or she would see Airplane 1 travel 90,000,000 kilometers and take five minutes to complete the journey. However, if he or she could see the clock in Airplane 1, the clock would have ticked off only 3.535 minutes, just as predicted by time dilation.

Airplane 1 makes the return trip in 3.535 minutes, also traveling only 63,600,000 kilometers. From Airplane 1's frame of reference, its round trip was 127,200,000 kilometers and took 7.07 minutes. From Airplane 2's frame of reference, Airplane

1's round trip was 180,000,000 kilometers and took 10 minutes. Both observers will see that Airplane 1's clock registered 7.07 minutes and Airplane 2's clock registered 10 minutes.




**Airplane 2's perspective**

From Airplane 2's frame of reference, Airplane 1 travels a total of 180,000,000 kilometers in five minutes. However, Airplane 1's clock has only registered 7.07 minutes. Airplane 1 is 2.93 minutes behind Airplane 2.

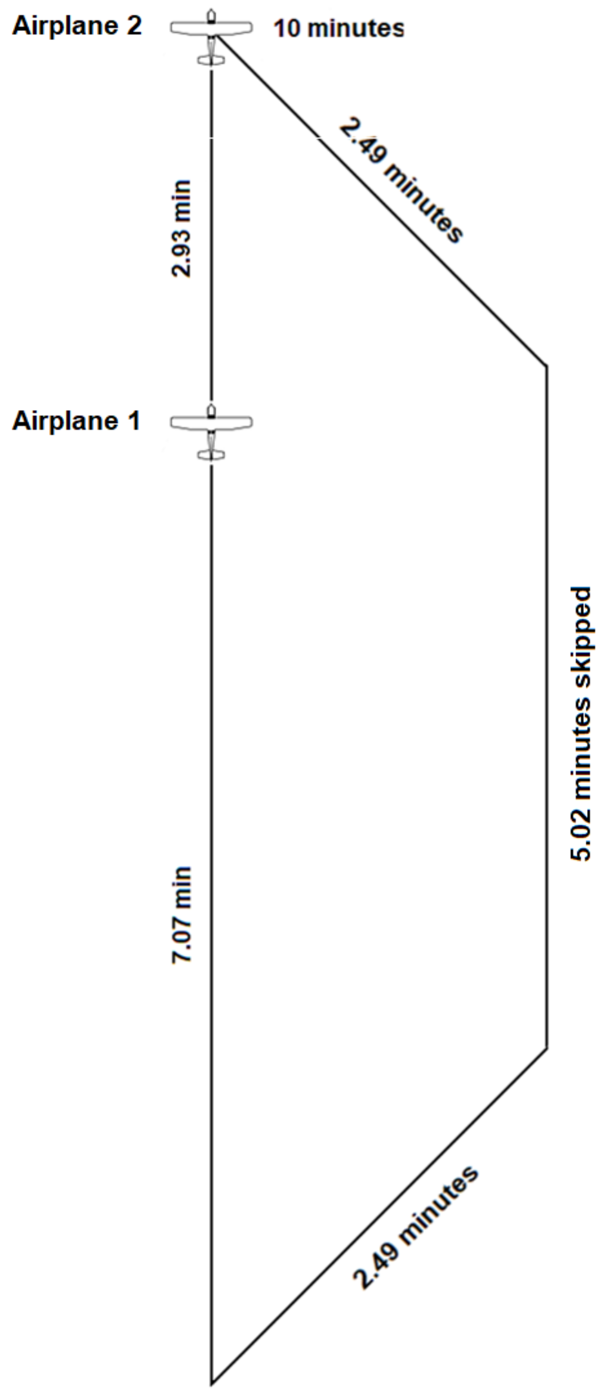
Now, let's look at how the observer in Airplane 1 sees Airplane 2. Remember that the observer in Airplane 1, the moving airplane, sees his or her airplane as stationary and everything else moving. Also, remember that the observer in

Airplane 1 sees space contracted where the observer in Airplane 2 does not. The observer in Airplane 1 sees Airplane 2 zip away in the opposite direction that Airplane 2 sees Airplane 1 go. However, since Airplane 1 sees space contracted, the observer in Airplane 1 sees Airplane 2 travel to a distance of only 63,600,000 kilometers. Since the observer in Airplane 1 also sees the clock in Airplane 2 running slower, he or she only sees Airplane 2's clock tick off 2.49 minutes during the trip.

Remember that when Airplane 1 reverses course, the observer in Airplane 1 sees Airplane 2 jump ahead in time. In this scenario, if the observer in Airplane 1 could see the clock in Airplane 2 in without propagation delay, he or she would see the clock jump ahead to 7.51 minutes.

Recall that Airplane 2 remains in one frame of reference, whereas Airplane 1 jumps from one frame of reference to another at the turnaround point. The clock in Airplane 2 is not jumping ahead. Instead, airplane 1 jumps from a frame of reference where the clock in Airplane 2 has registered 2.49 minutes to another frame of reference where the clock in Airplane 2 has already reached 7.51 minutes. The 5.02 minutes between are skipped when Airplane 1 jumps to the second frame of reference. If we don't want to imagine Airplane 1 jumping from one frame of reference to another, we can imagine Airplane 1's frame of reference rotating during the turnaround. In that case, the observer in Airplane 1 sees the clock in Airplane 2 speeding through 5.02 minutes in the time it takes for the turnaround. Nevertheless, from Airplane 1's frame of reference, Airplane 2 goes from 2.53 minutes behind in time to 2.53 minutes ahead during the turnaround.

Airplane 2 takes another 2.49 minutes to complete the round trip. The observer in Airplane 1 sees the clock in Airplane 2 register 2.49 minutes for the outbound leg, skip 5.02 minutes for the turnaround, and register another 2.49 minutes for the return leg. At the end of the trip Airplane 2's clock has registered 10 minutes. However, the clock in Airplane 1 has registered only 7.07 minutes.




The observer in Airplane 1 sees Airplane 2 zip away for 2.49 minutes and then zip back for another 2.49 minutes. However, Airplane 2's clock jumps ahead by 5.02 minutes at the turnaround point. When Airplane 2 returns, Airplane 1 has traveled only 7.07 minutes through time, but Airplane 2 has traveled 10 minutes through time. Airplane 1 finds itself 2.93 minutes behind Airplane 2.

To reiterate, the observer in Airplane 2 sees Airplane 1 zoom away to a point 90,000,000 kilometers away and back, taking 10 minutes for the round trip. However, the clock in Airplane 1 only registers 7.07 minutes. The observer in Airplane 1 only sees the trip as 63,600,000 each way and thus sees the trip take 7.07 minutes. Both observers agree that Airplane 1's clock shows 7.07 minutes and Airplane 2's clock shows 10 minutes. The observer in Airplane 1 sees Airplane 2 zip away to a point 63,600,000 kilometers away and return. The total trip takes 7.07 minutes by Airplane 1's clock. If the observer in Airplane 1 could see Airplane 2's clock, he or she would see the clock jump ahead by 5.02 minutes at the turnaround point. When Airplane 2 returns, Airplane 1's clock registers 7.07 minutes, but Airplane 2's clock registers 10 minutes. Both observers again agree that Airplane 1's clock registered 7.07 minutes and Airplane 2's clock registered 10 minutes. We had to account for both time dilation and length contraction to get everyone to agree.

**The twins paradox**

Notice that this solves the supposed twins paradox. The twins paradox presumes that there is a pair of twins. One twin rides a spaceship to some point in space and then returns, traveling at tremendous speed. Alpha Centauri is the usual destination at a distance of 4.37 light years. The other twin remains on Earth. Let's say the traveling twin travels at half the speed of light. At that speed, the traveling twin will age at 86.6 percent of the rate of the homebound twin. Therefore, after the round trip, the traveling twin will have aged only 15.1 years compared to the earthbound twin, who has aged 17.5 years.

The paradox appears to arise because the two frames of reference must be equivalent. Therefore, each twin should observe the other twin aging more slowly than him or herself. This corresponds to the original scenario when the airplanes are traveling apart, where each observer sees the other airplane falling behind. Since each twin must see the other aging more slowly, at the end of the round trip, each twin must be 2.34 years younger than the other. However, as we have already seen, when the traveling twin reverses direction, he or she jumps from a frame of reference, where the earthbound twin is behind in time, to another frame of reference, where the earthbound twin is ahead in time. As long as the earthbound twin remains stationary, the traveling twin can never catch up to the earthbound twin in time.

Let's look at this again to be sure we are following it. In the original scenario, when Airplane 1 turned back toward the original course, the observer in Airplane 1 saw Airplane 2 sweep around from behind to ahead. Airplane 1 could never make up the ground lost by deviating from the original course. In the second scenario, when Airplane 1 was pushed back toward Airplane two, its frame of reference rotated in spacetime from one where Airplane 2 was behind in time to a new frame of reference where Airplane 2 was ahead in time. Airplane 1 could never make up for the lost time as long as Airplane 2 didn't move.

In the twins paradox, when the traveling twin reverses course, his or her frame of reference rotates from one where the earthbound twin is behind in time to one where the earthbound twin is ahead in time. The traveling twin can never recover the lost time as long as the earthbound twin remains stationary. Each twin will see the other aging more slowly as long as they remain in inertial frames of reference, meaning frames of reference that are not changing velocity (speed or direction). When the traveling twin reverses direction, he or she is no longer in an inertial frame of reference but in an accelerating frame of reference. Once he or she settles on the new course back to Earth, he or she is in a different inertial frame of reference than the one he or she was in during the outbound leg of the journey. Therefore, the traveling twin ends up at a younger age than the stationary twin because he or she occupies two different inertial frames of reference during the trip.

Some say the solution to the twins paradox is that the traveling twin accelerates during the trip, whereas the earthbound twin doesn't. As we can see, this is correct because acceleration rotates your frame of reference. In each scenario, one observer accelerates, whereas the other doesn't. That acceleration rotates the associated frame of reference in space or spacetime. Therefore, acceleration changes your frame of reference to one where space and time don't line up with the original frame of reference. Hence, when one twin travels whereas the other doesn't, the traveling twin falls permanently behind the stationary twin in age by accelerating to reverse course.

Some others say that acceleration is unnecessary to solve the twins paradox. Their argument assumes each twin carries a clock. They then assume a second traveler is already on the return course with his or her own clock. The traveling twin and the second traveler pass at the turnaround point, where the second traveler synchronizes his or her clock with that of the traveling twin. When the second traveler arrives back at Earth, we find his or her clock is 2.34 years behind the stationary twin's clock. No acceleration required. This is also correct. The same two frames of reference are involved. The traveling twin accelerates to jump to the returning frame of reference. The second traveler is already there. It works either way.

**Conclusion**

In this discussion, I applied Euclidean geometry and relatable experiences to Einstein's theory of special relativity. As shown, everyday experience comes close to demonstrating time dilation. If the fourth dimension of time is equivalent to the three dimensions of space, time dilation must occur when one object moves relative to another. However, this results in a paradox if length contraction isn't also applied. Therefore, length contraction must also occur. Einstein made these postulations with nothing to back them other than they were compatible with experiment (such as the lack of results from the Michelson-Morley experiment). They are also compatible with the math developed by Lorentz and FitzGerald to quantify length contraction. Other than that, there is no reason to insist that time and space are equivalent. Math proves nothing unless experiment confirms it. However, modern experiments and practical application agree with Einstein's theories.